International Conference on Probability Theory and Mathematical Statistics

Dedicated to 100th Anniversary of Professor Gvanji Mania

Abstracts

July 16–18, 2018
Tbilisi, Georgia

Professor Gvanji Mania

Professor Gvanji Mania is a well-known Georgian mathematician and the founder of Georgian school of Probability Theory and Mathematical Statistics. He was born on May 29, 1918 in the village of Etsery to a family of a teacher. In 1935 he became a student of the faculty of Physics and Mathematics of the Tbilisi State University. In 1940-1945 he worked as an assistant in Pedagogical Institute of Zugdidi. In 1943-1946 he worked as an assistant at Railway Engineering Institute of Tbilisi. From 1945 to 1946 he was inspector of higher education school of the Ministry of Education of Georgia.

During 1946-1949 Mania was postgraduate student of Moscow Potemkin State Pedagogical Institute were he defended his thesis and received a candidate's degree. His scientific supervisor was a famous mathematician, professor N. Smirnov and official opponents were academician B. Gnedenko and professor Lyapunov. After the candidate's speech academician Gnedenko said: "Glivenko, Kolmogorov and Smirnov permanently pointed out the shortcomings of their theorems and it should be noted that G. Mania worked exactly on these interesting and important problems. The results obtained in the thesis have the first-class value". Professor Lyapunov said: "It is obvious that these results will be included into "gold fund" of mathematical statistics". No less successful was Mania's doctoral dissertation in 1963. His official opponents were academicians B. Khvedelidze, Y. Prokhorov and S. Syradjinov. In 1964 he was elected professor at Tbilisi State University. In 1949-1950 Professor Mania worked in Nikoloz Baratashvili Gory Pedagogical Institute as a docent. In 1950-1953 he was docent at Georgian Polytechnic Institute and in 1953-1956 senior researcher at Razmadze Tbilisi Mathematical Institute.

Professor Mania made a great contribution to the creation of new research centers in Georgia such as Computational Center and Institute of Applied Mathematics. In 1956-1964, Mania worked as deputy director in scientific field of Muskhelishvili Computational Center of Georgian Academy of Sciences and in 1966-1972 at Vekua Institute of Applied Mathematics. In 1968 Mania founded the Department of Probability Theory and Mathematical Statistics at the Tbilisi State University. He directed this department until the end of the life. Between 1973 and 1983 he was head of the Sector of Probability Theory and Mathematical Statistics at the Institute of Law and Economics and in 1983-1985 he was in charge of the same Sector at the Razmadze Mathematical Institute of the Georgian Academy of Sciences.

Professor Mania worked productively as a tutor and a researcher. He is the author of many scientific papers, textbooks and monographs. He was a member of various boards and societies, including that of the Bernoulli Society of International Statistical Institute since 1965 and member of the American Mathematical Society. He was also member of the board of International journal Statistics. In 1969, he was delegated to 37th session of International Statistical Institute in London and in 1970 he participated in International congress of mathematicians in Nice. He was awarded two government prizes and the Ivane Javakhishvili medal.

Professor Mania derived limiting distributions of several important statistics and tabulated these distributions. The proof of these statements is based on Abel and Tauber type theorems proved by Mania, where errors made in Feller's similar theorems were eliminated. These results became important for many scientists who used them in their research. These statistics are known as Mania's statistics in the scientific literature. In 1961 Professor Mania derived homogeneity criteria for two normal independent samples. Later, Mania investigated nonparametric estimation of the normal distribution density, and showed that it is impossible to generalize famous theorems of Boyd and Still. In 1974 the monograph "Statistical Estimation of Probability Distributions" was published, which was highly appreciated by peers. International Statistical Review cited 17 papers by Mania (including the above-mentioned monograph) in bibliography of Werts and Schneider on density estimation, where this monograph is mentioned as "an excellent book".

Professor Mania established the Georgian scientific school of probability theory and mathematical statistics, which covered a wide range of sub-disciplines. He made an invaluable contribution to the education of a new generation of Georgian mathematicians.

G. Mania died at the age of 67 years on March 16th, 1985.

Main Publications

(i) Monographs

1. Some methods of mathematical statistics. (Georgian) Publishing house of Georgian Academy of Sciences, Tbilisi, 1963, pp. 351.
2. Statistical estimation of probability distributions. (Russian) Tbilisi University Press, 1974, pp. 240.
3. Probability theory. (Georgian) Publishing house of Ministry of Education, Tbilisi, 1954, pp. 240.

4. Mathematical statistics in technics. (Georgian), Sabchota Sakartvelo, Tbilisi, 1958, pp. 345.
5. The course of probability theory. (Georgian) Tbilisi University Press, Tbilisi, 1962, pp. 340.
6. Linear programming. (Georgian), "Ganatleba", Tbilisi, 1967, pp. 295.
7. The course of high mathematic. (Georgian) Tbilisi State University , Tbilisi, 1967, pp. 498. (with P.Zeragia)
8. Ilia Vekua. (Georgian) Publishing house of Tbilisi State University, 1967, pp. 75. (with B.Hvedelidze)
9. Probability theory and mathematical statistic. (Georgian) Publishing house of Tbilisi State University, 1976, pp. 350.
10. A book of problems in probability theory and mathematical statistic. (Georgian). Publishing house of Tbilisi State University, 1976, pp. 120. (with A.Ediberidze and N.Anthelava)

(ii) Selected Publications

11. Generalization of A.N. Kolmogorov's criterion for the estimation of distribution laws by empirical data. (Russian) Dokl. Akad. Nauk SSSR, 69(1949), No. 4, 495-497.
12. Statistical estimation of distribution laws. (Russian) Uchenie Zapiski MGPI imeni V.P. Potiomkina 16 (1951), 17-63.
13. Practical applications of an estimation of a maximum of two-sided deviations of empirical distribution in a given interval of growth of a theoretical law. (Russian) Soobshch. AKad. Nayk GSSR,14(1953), No. 9, 521-524.
14. Practical applications of an estimation of a maximum of one-sided deviations of an empirical distribution in a given interval of growth of a theoretical law. (Russian) Proc. of Georgian Politechnical Institute, 30(1954), No. 9, 89-92.
15. Square estimation of divergence of normal densities by empirical data. (Georgian) Soobshch. AN GSSR, 17 (1956), No. 3, 201-204.
16. Square estimation of normal distribution densities by empirical data (Russian). Trydy Vsesojuznogo Mat. Siezda, IZD. AH SSSR, 1 (1956), 124-125.

17. Quadratic error of an estimation of twodimensional normal density by empirical data. (Russian) Soobshch. AN GSSR, 20 (1958), No. 6, 655-658.
18. Quadratic error of the estimation of normal density by empirical data. (Russian) Tr. Vychisl. Tsentra AH GSSR, 1 (1960), 75-96.
19. On one method of constructing of confidence regions for two samples from general population. (Russian) Soobshch. AN GSSR, 27 (1961), No. 2, 137-142.
20. Remark on non-parametric estimations of twodimensional densities. (Russian) Soobshch. AN GSSR, 27 (1961), No. 4, 385-390.
21. Square estimation of divergence of twodimensional normal distribution densities by empirical data. (Russian) Tr. BC AH GSSR, 2 (1961), 153-211.
22. Square estimation of divergence of twodimensional normal distribution densities by empirical data (Russian) Proc. of 6-th Vilnius Conference in Probab. Theory and Math. Stat., (1962), 407-409.
23. Quadratic error of an estimation of densities of normal distributions by two samples (Russian) Trudy. BC AH SSSR, 4 (1963), 213-216.
24. Hypothesis testing of identity of distributions of two independent samples. (Russian) Tr. Vichisl. Tsentra AH GSSR, 7 (1966), 1-34.
25. Square estimation of divergence of densities of multidimensional normal distribution by empirical data (po dannim viborki) (Russian) Proc. of Tbilisi State University,129 (1962), 373-382.
26. Quadratic error of the estimation of multidimensional normal distribution densities by empirical data. (Russian) Soobshch. AN GSSR, 52 (1968), No. 1, 27-30.
27. Quadratic error of the estimation of multidimensional normal distribution densities by empirical data (Russian) Probability Theory and Appl., 13 (1968), No. 2, 359-362.
28. Quadratic error of an estimation of densities of multidimensional normal distribution by empirical data. (Russian) Probability Theory and Appl., 14 (1969), No. 1, 151-155.
29. Quadratic error of an estimation of densities of multidimensional normal distribution by empirical data (Russian) Proc. of Tbilisi

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30. Quadratic error of an estimation of densities of multidimensional normal distribution by empirical data. Congres international des Mathematiciens, Nice, Paris,, (1970), Abstracts 260.
31. Quadratic error of an estimation of normal distribution densities by several samples (Russian) Soobshch. AN GSSR, 67 (1972), No. 2, 301-304.
32. One approximation of distributions of positive defined quadratic forms of normal random variables, (Russian).Soobshch. AN GSSR, 107 (1982), No. 2, 241-244.(with E.Khmaladze and V.Felker)
33. On the estimation of parameters of type of stable laws, Proceedings of the first International Tampere Seminar on linear Statistical Models and their Applications (1983) Tampere University, 1985, pp. 202-223. (with L.Klebanov and I.Melamed)
34. One problem of V.M.Zolotarev and analogue ofinfinitely divisible and stable distributions in the scheme of the sum of random number of random variables. (Russian) Probability Theory and Appl., 29 (1984), 757-760. (with L.Klebanov and I.Melamed)

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2. T.Shervashidze, Probability Theory and Mathematical Statistics (Russian) in 50-th anniversary of Tbilisi A.Razmadze Math. Institute, Metsniereba, Tbilisi, 1985.

Abstracts

Petre Babilua and Elizabar Nadaraya (Ivane Javakhishvili Tbilisi State University)

On some goodness-of-fit tests based on Wolverton-Wagner type estimates of distribution

Let $X_1, X_2, ..., X_n$ be a sequence of independent, identically distributed random variables, having a distribution density $f(x)$. Based on sample $X_1, X_2, ..., X_n$ it is required to test the hypothesis H_0 : $f(x) = f_0(x)$. Here we consider the hypothesis H_0 testing based on the statistic $T_n = na_n^{-1} \int (f_n(x) - f_0(x))^2 r(x)dx$, where $f_n(x)$ is the recurrent Wolverton-Wagner kernel estimate of probability density defined by $f_n(x) = n^{-1} \sum_{i=1}^{n} a_i K((a_i(x - X_i)))$, where a_i is an increasing sequence of positive numbers tending to infinity, $K(x), f_0(x)$ and $r(x)$ satisfy certain regularity conditions.

1. Question of consistency for the constructed criterion against any alternative $H_1 : f(x) = f_1(x)$, where $f_1(x)$ is such that $\int (f_n(x) - f_0(x))^2 r(x)dx > 0$, is studied.

2. The limiting behavior of the power is studied for a sequence of close to hypothesis H_0 alternatives of Pitman-Rosenblatt type [1] and it is shown that the tests based on T_n for abovementioned alternatives are more powerfull in limits than the tests of Bickel-Rosenblatt.

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[2]. Rosenblatt M. A quadratic measure of deviation of two-dimensional density estimates and a test of independence – Ann. Statist., 1975, v. 3, p. 1-14.

Denis Belomestny (Duisburg-Essen University)

Poisson deconvolution via Laplace transform

Besik Chikvinidze (Georgian Technical University, Institute of Cybernetics)

An extension of the mixed Novikov-Kazamaki condition

Given a continuous local martingale M, the associated stochastic exponential $E(M) = \exp\{M - \frac{1}{2}\langle M\rangle\}$ is a local martingale, but not necessarily a true martingale. To know whether $E(M)$ is a true martingale is important for many applications, e.g., if Girsanov's theorem is applied to perform a change of measure. We give a several generalizations of Kazamaki's results and finally construct a counterexample which does not satisfy the mixed Novikov-Kazamaki condition, but satisfies our conditions.

Ibrahim Didmanidze, Tatiana Fomina and Albert Shatashvili (Shota Rustaveli Batumi State University, Tugan-Baranovsky Donetsk National University)

On method of constructing algorithms for the efficient calculation of optimal estimates in extrapolation, filtration, and interpolation problems for functionals of random processes with values from a Hilbert space

Commonly, in studying of extrapolation, filtration and interpolation problems, the random processes are not fully or partially evaluated, as well as their full or partial participation and influence in some objects as arguments in functional dependence in form of functions or functionals of random processes , especially in case of incomplete (non complete) observations of these processes.

For example, suppose that a random process is considered as a vector from a finite-dimensional space and only a certain part of its components is non observable. Naturally, it is posed the problem of finding optimal estimates of non observable components of the random process or their functions and functionals in the above-listed problems.

These tasks are the most important in the theory of optimal estimates, and this problem is the subject of the proposed work.

In this paper observing some linear functional $g(\cdot)$ defined on H, we obtain general formulas for the effective calculation of optimal estimates for arbitrary $h(\cdot)$ functionals defined on some Hilbert space H, $\hat{h}(y(t))$ and represented the functional of random processes $y(t)$ with values in H. The behavior of $g(y(s))$ in the interval $0 \leq s \leq T$, at the point $T+\beta : t, s, T, T+\beta \in [0,a]$ is also purpose of the study, where $[0,a]$ is the domain of the random process $y(t)$. In the particular case when the process being studied is a solution of some nonlinear evolutionary differential equation, the obtained optimal estimates $\hat{h}(T+\beta)$ decompose at the point $T+\beta$ in a small parameter powers, which the equation contains at small nonlinearity.The coefficients of the decomposition are given in the form of algorithms and are calculated explicitly via the known values of the differential equation itself. In the most general formula of the optimal estimate, $\hat{h}(T+\beta)$ contains all three estimates and all depends on the choice of the functionals $h(\cdot)$, $g(\cdot)$ and the parameter β. For $\beta > 0$, $\beta = 0$ and $\beta < 0$, respectively the optimal extrapolation, optimal filtering problem and optimal interpolation are solved.

Besarion Dochviri, Vakhtang Jaoshvili and Zaza Khechinashvili (Ivane Javakhishvili Tbilisi State University)

On the optimal stopping with incomplete data

The problem of optimal stopping of a partially observable stochastic process in Kalman-Bucy's model is reduced to the problem of optimal stopping of a complete observable stochastic process.The convergence of corresponding payoffs is proved when $\varepsilon_1 \to 0, \varepsilon_2 \to 0$, where $\varepsilon_1, \varepsilon_2$ are small perturbation parameters of stochastic process.

Lasha Ephremidze (New York University Abu Dhabi, A. Razmadze Mathematical Institute (TSU))

Granger causality and matrix spectral factorization with applications in neuroscince

Granger causality [5] is a statistical tool providing a numerical estimation of whether the process generating stationary time series $\{Y_k\}_{k\in\mathbb{Z}}$ causes another process of generating stationary time series $\{X_k\}_{k\in\mathbb{Z}}$. By definition, we have such causality if the inclusion of the known values of Y in the linear prediction of X significantly reduces its mean square error, i.e.

$$\min_{\alpha}\|X_0-\sum_{k>0}\alpha_k X_{-k}\| << \min_{\alpha,\beta}\|X_0-\sum_{k>0}\alpha_k X_{-k}-\sum_{k>0}\beta_k Y_{-k}\|.$$

Expanding these ideas, Geweke [4] has defined a frequency dependent causality for more than two processes involved. Dhamala et al [1] have shown that computation of such causality is reduced to factorization of power spectral density matrix of a stationary time series, which is known as non-parametric method of Granger causality computation.

Since then, Wilson's matrix spectral factorization algorithm [6] has been widely utilized by the neuroscince community in order to estimate functional causalities of different brain area activities, analyzing multichannel EEG recordings. However, if power spectral density matrix is degenerated or ill-conditioned at some isolated frequency points in the domain, then Wilson's algorithm fails to perform an accurate factorization. As it is mentioned in [3], such singular situations are common in the study of unstable biological systems.

In the present talk, we would like to demonstrate that the novel matrix spectral factorization algorithm [2] is capable of overcoming above mentioned obstacles. Thus, this algorithm leads to new opportunities for accurate estimation of frequency dependent Granger causality, for complex systems containing unstable modes.

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Shakir Formanov (Uzbekistan Academy of Sciences, V.I.Romanovskiy Institute of Mathematics)

On new versions of the Lindeberg-Feller's limit theorem

It is well known that the classical Lindeberg condition is sufficient for validity of the central limit theorem. It will be also a necessary if the summands satisfy the condition of infinite smallness (Feller's theorem). The limit theorems for the distributions of the sums of independent random variables which do not use the condition of infinite smallness were called non-classical.

In this paper a non-classical version of Lindeberg-Feller's theorem is given. The exact bounds for the Lindeberg, Rotar characteristics using the difference of the distribution of sum of independent random variables and a standard normal distribution are established. These results improve Feller's theorem.

Karen Gasparyan (Yerevan State University)

The optional strong martingales and supermartingales

The concept of optional strong supermartingales has been intoduced by Mertens in [1], where the Doob-Meyer type decomposition for such supermartingales has been obtained. At the same time several works on the general theory of random processes in which the so-called “usual" conditions are not required (Dellacherie and Meyer (1975, 76), Dellacherie (1976), Galchuk (1977, 80, 84), Horowitz (1978)) have appeared. The stochastic calculus for laglad optional strong martingales in this case was constructed (Lenglart (1980), Galchuk (1985)) and the Doob-Meyer decomposition for optional strong supermartingales was obtained (Galchuk (1981), Dellacherie and Meyer (1982)).

Subsequently the research in this direction was continued by Galchuk, Kudrjavcev and by the author, and statistical applications of this theory were obtained by the author. The uniform variant of the Doob-Meyer decomposition for optional strong supermartingales in the “usual" case, which is substantially used in financial mathematics, was established by Kramkov in [2] (see also [3]). When the “usual" conditions are not satisfied, a similar result was obtained in [4]. Recently, interest in the theory of optional strong martingales and supermartingales reappeared in connection with their applications in financial mathematics (Kuhn and Stroh (2009), Czichowsky and Schachermayer (2014, 16), Abdelghani and Melnikov (2016, 17), Libo (2018)).

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George Giorgobiani and Vakhtang Kvaratskhelia (Georgian Technical University, Niko Muskhelishvili Institute of Computational Mathematics)

Maximal inequalities and their applications to orthogonal and Hadamard matrices

Using Hoeffding-Chernoff bound, maximal inequalities for signed vector summands and corresponding probabilistic estimates are established. By use of "transference technique", appropriate maximal inequalities are derived for permutations. An application to orthogonal and Hadamard matrices is considered.

Shota Gugushvili (Leiden University)

Fast and scalable non-parametric Bayesian inference for Poisson point processes

We study the problem of non-parametric Bayesian estimation of the intensity function of a Poisson point process. The observations are assumed to be n independent realisations of a Poisson point process on the interval $[0, T]$. We propose two related approaches. In both approaches we model the intensity function as piecewise constant on N bins forming a partition of the interval $[0, T]$. In the first approach the coefficients of the intensity function are assigned independent Gamma priors. This leads to a closed form posterior distribution, for which posterior inference is straightforward to perform in practice, without need to recourse to approximate inference methods. The method scales extremely well with data. On the theoretical side, we prove that the approach is consistent: as $n \to \infty$, the posterior distribution asymptotically concentrates around the "true", data-generating intensity function at the rate that is optimal for estimating h-Hölder regular intensity functions ($0 < h \leq 1$), provided the number of coefficients N of the intensity function grows at a suitable rate depending on the sample size n.

In the second approach it is assumed that the prior distribution on the coefficients of the intensity function forms a Gamma Markov chain. The posterior distribution is no longer available in closed form, but inference can be performed using a straightforward version of the Gibbs sampler. We show that also this second approach scales well.

Practical performance of our methods is first demonstrated via synthetic data examples. It it shown that the second approach depends in a less sensitive way on the choice of the number of bins N and outperforms the first approach in practice. Finally, we analyse three real datasets using our methodology: the UK coal mining disasters data, the US mass shootings data and Donald Trump's Twitter data.

Joint work with Frank van der Meulen, Moritz Schauer and Peter Spreij.

Jevgenijs Ivanovs (Aarhus University)

Discretization error for the supremum of a Lévy process

Lévy processes are used extensively in financial mathematics and insurance risk. In particular, the supremum of a Lévy process presents some major interest – think of pricing barrier options, calculating exceedance or ruin probabilities, and so forth. There are, however, few examples where the law of the supremum is available in explicit form, and an obvious way to evaluate this law is to perform Monte Carlo simulation by sampling the Lévy process on an equidistant grid. In this talk I will present limit theory for the corresponding discretization error, which is based on the zooming-in concept and an invariance principle. Additionally, I will discuss connections to high-frequency statistics and provide some further applications.

Revaz Kakubava and Ilia Mikadze (Georgian Technical University)

An alternative transient analysis for semi-Markov queuing systems

In the talk, using purely probabilistic argumentation, theorems are presented, which, on one hand simplify the analysis of $M/G/1$ queuing systems using the auxiliary variables method. On the other hand, it allows not using partial differential equations infinite system at all in non classical boundary value problem of mathematical physics, and directly yields the system solution in terms of operational calculus.

Estate Khmaladze (Victoria University of Wellington)

The colour blind problem

Consider an experiment when one observes pairs of balls with random diameters $(X_i, Y_i)_{1 \leq i \leq n}$. In each pair, one ball, with diameter X_i, is green, while the other ball, with diameter Y_i, is blue. The pairs $(X_i, Y_i)_{1 \leq i \leq n}$ are i.i.d., and within each pair X_i and Y_i are independent. We want to test that diameters of green and blue balls have the same distribution. Denoting these distributions by P_1 and P_2, we want, therefore, to test non-parametric hypothesis

$$H_0 : \; P_1 = P_2 \text{ (and equal to some unspecified } Q).$$

As it is formulated so far, the problem is a classical one and the class of test statistics is well understood. Namely, if P_{1n} and P_{2n} are empirical distribution functions based on $\{X_i\}_{1 \leq i \leq n}$ and $\{Y_i\}_{1 \leq i \leq n}$, respectively, any statistic based on the empirical process

$$\sqrt{n}(P_{1n} - P_{2n}), \text{ or } \sqrt{n}[P_{1n} - \frac{1}{2}(P_{1n} + P_{2n})],$$

invariant under time transformation $Q(x) = t$, have distribution which is the same for all Q. If we are interested in change only between expected values of diameters of green and blue balls, then just Student's statistic $\sqrt{n}(\bar{X}_n - \bar{Y}_n)$ (with proper normalisation) will provide a good test.

However, what can one do in the case when an observer cannot distinguish the colours of the balls, that is, when he is "colour blind"?

In this case, it is not possible to construct P_{1n} and P_{2n}, or even averages $\bar{X}_n$ and $\bar{Y}_n$. Yet, we show that systematic approach to testing H_0 is possible, again providing an empirical process and test statistics, based on this process, with distribution independent of (unknown) common Q and explains what is the price one pays for "colour blindness" in terms of the power of our tests.

Joint work with Laura Dumitrescu.

Lev B. Klebanov (Charles University)

G.M. Maniya and the Theory of Sums of a Random Number of Random Variables

In the talk there is given a survey of G.M. Maniya's work on the theory of summation of random number of random variables. Some new results obtained in this theory are represented. We also discuss possible applications of this theory to modeling of dielectric relaxation.

Nanuli Lazrieva and Temur Toronjadze (A. Razmadze Mathematical Institute, Georgian-American University)

Recursive estimation of one-dimensional parameter of Compound Poisson process

Recursive estimation procedure of a one-dimensional parameter of Levy measure of Compound Poisson process is introduced and their asymptotic properties are investigated.

Hans Rudolf Lerche (University of Freiburg)

A measure transformation approach to optimal stopping

This presentation shows an approach to Optimal Stopping which uses the following idea. Find a representation of the expected gain as

$$R(\tau) = \int g(X_\tau) M_\tau I_{\{\tau<\infty\}} dP,$$

where M_t, $t \geq 0$, is a positive martingale under P of the underlying process X_t, $t \geq 0$, g is a function with unique maximum or minimum and τ is a stopping time. An optimal stopping time stops at argmax (or argmin). In the case of exponentially discounted diffusions the optimal stopping set can be characterized by this approach. The method can also be extended to stopping games to find the Nash-equilibria.

This is joint work with M. Beibel and D. Stich.

Badri Mamporia (Georgian Technical University, Niko Muskhelishvili Institute of Computational Mathematics)

The Itô formula for the Itô processes driven by the cylindrical Wiener process in a Banach space

In the space of generalized random elements we consider the general Itô processes driven by the cylindrical wiener process and prove the Ito formula for them. Afterward, from the main Itô process in a Banach space driven by the cylindrical Wiener process, we produce the generalized Itô process in the space of generalized random elements and write the Itô formula already proven for this case. Then we check radonizability of the members of the obtained equality and as they turn out Banach space valued, we get the Itô formula in an arbitrary Banach space. Earlier we implemented this approach in some special cases.

Joint work with Omar Purtukhia.

Badri Mamporia and Omar Purtukhia (Georgian Technical University, Niko Muskhelishvili Institute of Computational Mathematics and Ivane Javakhishvili Tbilisi State University, A. Razmadze Mathematical Institute)

On functionals of the Wiener process in a Banach space

In development of stochastic analysis in a Banach space, one of the main problems is to establish existence of a stochastic integral from predictable Banach space-valued (operator-valued) random process. In the problem of representation of a Wiener functional as a stochastic integral we are faced with an inverse problem: we have a stochastic integral as a Banach space-valued random element, and we are looking for a suitable predictable integrand process. There are positive results only for a narrow class of Banach spaces with special geometry (UMD Banach spaces). We consider this problem in a general Banach space for a Gaussian functional.

Sherzod Mirakhmedov (Institute of Mathematics of Uzbekistan Academy of Sciences)

The Probabilities of Large Deviations Associated with Multinomial Distribution and Asymptotical Efficiencies of Certain Class of Goodness of Fit Tests

Robert Mnatsakanov (West Virginia University)

Nonparametric density estimation via the scaled Laplace transform inversion

In this talk the scaled values of estimated Laplace transform of the underlying distribution function are used to construct the estimate of corresponding density function a positive random variable. Asymptotic expressions of the bias term and the mean squared errors are derived. By means of graphical illustrations and the values of the average L_2-errors we conducted comparisons of the finite sample performances of proposed estimate with those based on traditional kernel density approach.

Joint work with Fairouz Elmagbri.

Rimas Norvaiša (Vilnius University)

Convergence in law of partial sums of linear processes in p-variation norm

Let $X_1, X_2, \dots$ be a sequence of short memory linear processes, S_n be the n-th partial sum process $S_n(t) = X_1 + \cdots + X_{\lfloor nt \rfloor}$, $t \in [0,1]$, and $2 < p < \infty$. We shall discuss a convergence in law of $n^{-1/2}S_n$ to a Wiener process in p-variation norm. In the case when $X_1, X_2, \dots$ is a sequence of independent identically distributed real-valued random variables, the result is proved in [1].

Joint work with A. Račkauskas.

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Mark Podolskij (Aarhus University)

Estimation of the linear fractional stable motion

In this talk we investigate the parametric inference for the linear fractional stable motion in high and low frequency setting. The symmetric linear fractional stable motion is a three-parameter family, which constitutes a natural non-Gaussian analogue of the scaled fractional Brownian motion. It is fully characterised by the scaling parameter $\sigma > 0$, the self-similarity parameter $H \in (0,1)$ and the stability index $\alpha \in (0,2)$ of the driving stable motion. The parametric estimation of the model is inspired by the limit theory for stationary increments Levy moving average processes that has been recently studied in the paper "Power variation for a class of stationary increments Lévy driven moving averages" (Annals of Probability 45(6B), 4477–4528).

Igor Prünster (Bocconi University)

Discrete structures and prediction in Bayesian Nonparametrics

Discrete random probability measures and the exchangeable random partitions they induce are key tools for addressing a variety of estimation and prediction problems in Bayesian inference. In this talk we focus on the family of Gibbs-type priors, a recent elegant and intuitive generalization of the Dirichlet and the Pitman-Yor process priors. Several distributional properties are presented and their implications for Bayesian nonparametric inference highlighted. Illustrations in the contexts of mixture modeling, species sampling and curve estimation are provided.

Marina Santacroce (Politecnico di Torino)

Forward backward semimartingale systems for utility maximization

In this talk we discuss a method which gives a constructive characterizations of the optimal strategies for expected utility maximization problems for a non Markovian setting. The proposed method seems to provide an alternative to dynamic programming and works for general utilities.

Joint work with B. Trivellato.

Malkhaz Shashiashvili (Ivane Javakhishvili Tbilisi State University)

On the integral relationship between the early exercise boundary and the value function of the American put option

We prove a new integral relationship between the American put option early exercise boundary and its value function in the generalized Black-Sholes model. Having at hand any uniform approximation of the American option value function it is possible to construct the L_2 approximation to the unknown early exercise boundary without additional assumption on the second order partial derivative of the value function to be bounded below by some positive constant.

Sergey N. Smirnov (National Research University Higher School of Economics)

Feller transition kernel with supports of measures defined by a multivalued function

Let X and Y be topological spaces with some proper regularity properties, equipped with Borel σ-algebras $\mathcal{B}_X$ and $\mathcal{B}_Y$, respectively; $P(x, B)$ be stochastic transition kernel (i.e. the mapping $x \mapsto P(x, B)$ is measurable for all $B \in \mathcal{B}_Y$, and the mapping $B \mapsto P(x, B)$ is a probability measure for any $x \in X$); $supp(P(x, \cdot))$ be topological support of a measure $B \mapsto P(x, B)$. If a transition kernel $P(x, B)$ satisfies the Feller property (that is, the mapping $x \mapsto P(x, \cdot)$ is continuous (with respect to the weak topology on the space of probability measures), then the multivalued mapping $x \mapsto supp(P(x, \cdot))$ is lower semicontinuous. Conversely, consider a multivalued mapping $x \mapsto S(x)$, where $x \in X$ and $S(x)$ is a non-empty closed subset of Polish space Y. If $x \mapsto S(x)$ is lower semicontinuous, then there are Feller transition kernels such that $supp(P(x, \cdot)) \subseteq S(x)$ for all $x \in X$; moreover, there exist Feller transition kernels such that $supp(P(x, \cdot)) = S(x)$ for all $x \in X$.

Vazha Tarieladze (Georgian Technical University, Niko Muskhelishvili Institute of Computational Mathematics)

On two inequalities of Nikishin

We will discuss a proof of inequalities obtained by Nikishin ([1], [2]) based on Chobanyan's transference lemma.

Joint work with Sergei Chobanyan.

References

[1] E. M. Nikishin. Convergent rearrangements of series of functions. *Mat. Zametki*, 1967, Volume 1, Issue 2, 129–136.

[2] E. M. Nikishin. Rearrangements of series in L_p. *Mat. Zametki*, 1973, Volume 14, Issue 1, 31–38.

Mamuli Zakradze (Georgian Technical University, Niko Muskhelishvili Institute of Computational Mathematics)

The method of probabilistic solution for 3D Dirichlet ordinary and generalized harmonic problem in finite domains bounded with one surface

The Dirichlet ordinary and generalized harmonic problems for some 3D finite domains are considered. The term "generalized" indicates that a boundary function has a finite number of first kind discontinuity curves. An algorithm of numerical solution by the method of probabilistic solution (MPS) is given, which in its turn is based on a computer simulation of the Wiener process. Since, in the case of 3D generalized problems there are none exact test problems, therefore, for such problems, the way of testing of our method is suggested. For examine and to illustrate the effectiveness and simplicity of the proposed method five numerical examples are considered on finding the electric field. In the role of domains are taken ellipsoidal, spherical and cylindrical domains and both the potential and strength of the field are calculated. Numerical results are presented.

Joint work with Badri Mamporia, Murman Kublashvili, Nana Koblishvili.

Zurab Zerakidze and Mamuli Mumladze (Gori State University)

Linear consistent criteria for testing hypotheses

In this talk we prove a theorem that indicates when statistical structure [1] admits unbiased consistent criteria for testing hypotheses. Let H be a separable Hilbert space,$B(H)$ the Borel σ-algebra of subsets in H. Let a family of probability measures $(\mu_h,\ h \in H)$ be given on $B(H)$, that have different means $(a_h, z) = \int (z, x)\mu_h(dx)$ for all $z \in H$, and the same correlation operator B. We shall assume that the hypotheses are these average values. Let $\Theta \subset H$ be the set of hypotheses, which is assumed to be a convex manifold in H. Let $H_n \subset H_{n+1}$ be some increasing sequence of subspaces of H, such that $\bigcup\limits_n H_n$ is everywhere dense in H. As a criterion, we consider a sequence of linear mappings $\delta_n : H_n \longrightarrow \Theta$, $\delta_n(x) = (b_n, x)$, where $b_n \in H_n$.

Theorem. Let P_n be the projector on H_n. Operator $P_n^{'}$ satisfies the following conditions: $P_n^{'}(u) = u$, if u belongs to the orthogonal complement $H_n - P_n(H)$ of $P_n(H)$ in H_n and $\overline{P_n^{'}}(\nu) = H_n - P_n(H)$ for all $\nu \in H$, $P_n^{'}B = BP_n^{'}$ on H_n. If $z \in \overline{\bigcup_n H_n}$, then in order for the statistical structure to allow an unbiased consistent criteria of testing hypotheses harmonized with the sequence H_n, it is necessary and sufficient that $\lim n \longrightarrow \infty(BP_n^{'}(z), P_n P_n^{'}(z))$.

References

[1] L. Aleksidze, M. Mumladze, Z. Zerakidze. The consistent criteria of hypotheses. *Modern Stocastics: Theory and Appl.*. Vol 1, 2014.

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