On Some Goodness-of-Fit Tests Based on Wolverton-Wagner Type Estimates of Distribution Density

P. Babilua*, E. Nadaraya**

Affiliation: *Iv. Javakhishvili Tbilisi State University, Faculty of Exact and Natural Sciences, Department of Mathematics, Tbilisi, Georgia;
**Iv. Javakhishvili Tbilisi State University, Faculty of Exact and Natural Sciences, Department of Mathematics, Tbilisi, Georgia

E-mail: *elizbar.nadaraya@tsu.ge; **petre.babilua@tsu.ge

Let $X_1, X_2, ..., X_n$ be a sequence of independent, equally distributed random variables, having a distribution density $f(x)$. Based on sample $X_1, X_2, ..., X_n$ it is required to check the hypothesis

$$H_0 : f(x) = f_0(x).$$

here we consider the hypothesis H_0 testing, based on the statistics

$$T_n = na_n^{-1} \int (f_n(x) - f_0(x))^2 r(x) dx,$$

where $f_n(x)$ is the recurrent Wolverton-Wagner kernel estimate of probability density defined by:

$$f_n(x) = n^{-1} \sum_{i=1}^{n} a_i K((a_i(x - X_i))),$$

where a_i is an increasing sequence of positive numbers tending to infinity, $K(x), f_0(x)$ and $r(x)$ satisfy certain regularity conditions.

1. Question of consistency for the constructed criterion against any alternative $H_1 : f(x) = f_1(x)$, where $f_1(x)$ is such that $\int (f_n(x) - f_0(x))^2 r(x) dx > 0$ is studied.

2. The limiting behavior of the power is studied for sequence of close to hypothesis H_0 alternatives of type Pitmen and Rosenblatt [1] and it is shown that the tests based on T_n for above mentioned alternatives are more powerfull in limits than the tests based of Bickel-Rosenblatt [2].

References

1. Bickel P. J., Rosenblatt M. On some global measures of the deviations of density function estimates – Ann. Statist., 1973, v. 1, p. 1071-1095.
2. Rosenblatt M. A quadratic measure of deviation of two-dimensional density estimates and a test of independence – Ann. Statist., 1975, v. 3, p. 1-14.