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Feller transition kernel with supports of measures defined by a multivalued function

Abstract

Let X and Y be topological spaces with some proper regularity properties, equipped with Borel σ-algebras $\mathcal{B}_X$ and $\mathcal{B}_Y$, respectively; $P(x, B)$ be stochastic transition kernel (i.e. the mapping $x \mapsto P(x, B)$ is measurable for all $B \in \mathcal{B}_Y$, and the mapping $B \mapsto P(x, B)$ is a probability measure for any $x \in X$); $supp(P(x, \cdot))$ be topological support of a measure $B \mapsto P(x, B)$. If a transition kernel $P(x, B)$ satisfies the Feller property (that is, the mapping $x \mapsto P(x, \cdot)$ is continuous (with respect to the weak topology on the space of probability measures), then the multivalued mapping $x \mapsto supp(P(x, \cdot))$ is lower semicontinuous. Conversely, consider a multivalued mapping $x \mapsto S(x)$, where $x \in X$ and $S(x)$ is a non-empty closed subset of Polish space Y. If $x \mapsto S(x)$ is lower semicontinuous, then there are Feller transition kernels such that $supp(P(x, \cdot)) \subseteq S(x)$ for all $x \in X$; moreover, there exist Feller transition kernels such that $supp(P(x, \cdot)) = S(x)$ for all $x \in X$.