An extension of the mixed Novikov-Kazamaki condition

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Abstract Given a continuous local martingale M, the associated stochastic exponential $\mathcal{E}(M) = \exp\{M - \frac{1}{2}\langle M\rangle\}$ is a local martingale, but not necessarily a true martingale. To know whether $\mathcal{E}(\mathcal{M})$ is a true martingale is important for many applications, e.g., if Girsanov's theorem is applied to perform a change of measure. We give a several generalizations of Kazamaki's results and finally construct a counterexample which does not satisfy the mixed Novikov-Kazamaki condition, but satisfies our conditions.

Now we formulate the main result of this paper:

Theorem Let a_s be a predictable, M-integrable process and let φ be a lower function such that the following conditions hold:
(i) $|a_s - 1| \geq \varepsilon$ for some $\varepsilon > 0$;
(ii) $\varphi(x)$ can be represented as a sum of non decreasing and bounded functions $\varphi(x) = f(x) + g(x)$;

$$(iii) \quad D = \sup_{0\leq\tau\leq T} E\exp\left\{\int_0^\tau a_s dM_s + \int_0^\tau \left(\frac{1}{2} - a_s\right) d\langle M\rangle_s - \varepsilon\varphi(\langle M\rangle_\tau)\right\} < \infty$$

where *sup* is taken over all stopping times. Then the stochastic exponentials $\mathcal{E}(\int adM)$ and $\mathcal{E}(M)$ are uniformly integrable martingales.